Flexure beams and slabs (rectangular cross-section)
– Doubly reinforced section –

Ref. EN 1992-1-1
Sec. 3.1.7(3)
Exp. (3.19) – (3.21)
Sec. 5.5

Geometry cross-section
Rectangular shape:
Width b = 350 mm
Height h = 300 mm

Analysis
Design moment: M_{Ed1} = 150,00 kNm
Design shear (for additional tensile force only): V_{Ed} = 250 kN
$\Delta M_{Ed} = \Delta F_{td}\, z = 0.5 V_{Ed}(\cot\theta - \cot\alpha)\, z$ = 63,98 kNm

Sec. 6.2.3(7) - Eq. 6.18

Design moment (additional tensile force due to shear):
$M_{Ed} = M_{Ed1} + \Delta M_{Ed}$ = 213,98 kNm
(assuming – 20% moment redistribution: δ = 0,80).

Exp. (6.18) applied

Materials – characteristic/design strengths
Concrete
Strength class for concrete:

Table 3.1

f_{ck} = 35 N/mm^2 = 35 MPa
$f_{cd} = \alpha_{cc} f_{ck}/\gamma_c = 0,85 f_{ck}/1,50$ = 19,83 N/mm^2

Steel
Strength of reinforcement (Class B, C):

Table C.1 – Annex C

f_{yk} = 500 N/mm^2 = 500 MPa

$f_{yd} = f_{yk}/\gamma_s = f_{yk}/1,15 = 0,87 f_{yk}$ = 435 N/mm^2 = constant
E_s = 200 GPa; f_{yd}/E_s = 0,002174 [-].

Cl. 3.2.7(2) b)

Beam lever arm
For grades of concrete up to C50/60, ε_{cu3} = 3,5/1000, η = 1, λ = 0,8.

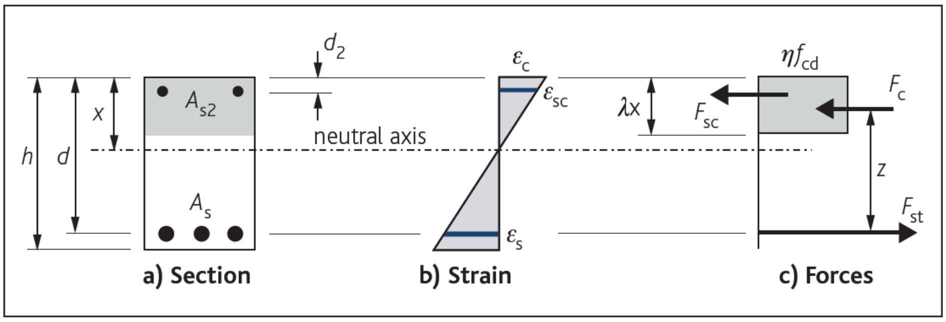

Figure 3.5 – modified

Durability and cover to reinforcement

Nominal cover:

$c_{nom} = c_{min} + \Delta c_{dev} = 25$ mm — **Exp. (4.1)**

Maximum size of aggregate: $d_g = 20$ mm

Effective depth:

$d = h - c_{nom} - \phi_{link} - 0{,}5\phi_{vert,1} = (300 - 25 - \phi_{link} - 0{,}5\phi_{vert,1}) = 227{,}5$ mm — **Cl. 8.2(2) – Note**

where:

ϕ_{link} is the diameter of the shear reinforcement

$\phi_{vert,1}$ equivalent vertical dimension (MS tensile bars in N $\geq$ 1 layers).

Depth to centroid of compressive reinforcement from compression fiber: — **Figure 5.5 (modified)**

$d_2 = 46$ mm $= c_{nom.} + \phi_{link} + 0{,}5\phi_{vert,2}$.

Linear elastic analysis with limited redistribution

Sec. 5.5

Ratio of the redistributed moment to the elastic bending moment:

$\delta = 0{,}80 \geq k_1 + k_2 x_u/d$ (for $f_{ck} \leq 50$ MPa) — **Exp. (5.10a)**

$\delta = 0{,}80 \geq 0{,}4 + [1\cdot(0{,}6 + 0{,}0014/\varepsilon_{cu2})]x_u/d$ (for $f_{ck} \leq 50$ MPa)

$\delta = 0{,}80 \geq k_5 = 0{,}7$ (for reinforcement Class B and C).

With $\varepsilon_{cu2} = 3{,}5/1000$ (for $f_{ck} \leq 50$ MPa): — **Table 3.1**

$x_u/d = 0{,}400$ (for linear elastic analysis with limited redistribution)

$K' = 0{,}453\cdot(1 - 0{,}4\cdot x_u/d)\, x_u/d = 0{,}152$.

Compressive steel

$d_2 = 46$ mm $< x_u = 91$ mm (satisfactory).

$\gamma_2 = d_2/d = 0{,}202$ [-].

Let

$$\alpha_{s2} = \frac{\sigma_{s2}}{0{,}87 f_{yk}}$$ within the range (0; 1] ($\sigma_{s2} = 0{,}87 f_{yk}$ for α_2):

$$\alpha_{s2} = \frac{E_s}{0{,}87 f_{yk}} \cdot \frac{0{,}0035}{x_u/d} \cdot (x_u/d - \gamma_2) < 1{,}00 \rightarrow \alpha_{s2} = 0{,}80 \text{ [-]}$$

From the proportion of the strain distribution diagram.

$$\frac{0{,}0035}{x_u/d} = \frac{0{,}0035 - 0{,}87 f_{yk}/E_s}{\gamma_{2,lim,el}} \rightarrow \gamma_{2,lim,el} = 0{,}152 \text{ [-]}$$

$\sigma_{s2} = \alpha_{s2} 0{,}87 f_{yk} = 346$ MPa (compressed steel not yielded: $\gamma_2 > \gamma_{2,lim,el}$).

Flexural design

$K = M_{Ed}/(bd^2 f_{ck}) = 0{,}338 > K' = 0{,}152$

(provide compression reinforcement).

For $d > 0{,}8x_u$:

using K' instead of K:
$z/d = 0{,}5 \cdot [1 + (1 - 3{,}529 \cdot K')^{0{,}5}] = 0{,}840$ (z/d limited to a max of 0,95):
$z = 0{,}840d = 191{,}1$ mm (beam lever arm)

Area of the compression reinforcement required

Cl. 9.2.1.1 – Note

$A_{s2} = (K - K')f_{ck}bd^2/[\sigma_{s2} \cdot (d - d_2)] = 1870 \text{ mm}^2$
► [Try 5H22 x 1 layers. Single layer with: H22 @ 65 mm)]

Percentage area of the compression reinforcement provided

Cl. 9.2.1.1(3) – Note

$\rho_{s2} = 100 \cdot A_{s2}/(bd) = 2{,}39\% \leq 4bh/100$ (satisfactory) $= A_{s,max}$

Area of tension reinforcement required

Cl. 9.2.1.1 – Exp. (9.1N)

With $z = 191{,}1 \text{ mm} \leq 0{,}95d$
$A_s = K'f_{ck}bd^2/(0{,}87 \cdot f_{yk}z) + (K - K')f_{ck}bd^2/[0{,}87f_{yk}(d - d_2)] = 2649 \text{ mm}^2$
$A_s \geq A_{s,min}$ (satisfactory)
► [Try 3H25 x 2 layers. Single layer with: H25 @ 128 mm)]

Percentage area of the tensile reinforcement provided

Cl. 9.2.1.1(3) – Note

$\rho_s = 100 \cdot A_s/(bd) = 3{,}70\% \leq 4bh/100$ (satisfactory) $= A_{s,max}$.